Exercise #7

Trajectory generation and tracking

P7.1
Consider the linear PID position controller given in CH6_Fig.2 (*given within CH6 Exercises at the bottom of the document*), with proportional and derivative actions replaced into the feedback path. Assume that the reference profile has a ramp shape, with the position reference samples $\theta^*_k = kR^*$. Calculate the steady-state position error $\Delta\theta(\infty)$. Calculate this error for the case when the integral and proportional gains are in the direct path, while the derivative gain resides in the feedback path.

P7.2
Consider the simplified Simulink model of a linear discrete-time PID position controller, given in Fig. 7.2, where the parameters K_{FB} and K_M are assumed to be equal to one. The reference profile has a ramp shape, incrementing at a constant slope. The proportional action can be switched from the direct path into the feedback path. Run the model for both locations of the proportional gain. Plot the output error and output position, verify the results from P7.1/S7.2, and note the overshoot in the output position. Note that Matlab files *P7_2cmd.m*, *P7_2F.mdl*, and *P7_2D.mdl* can be used to obtain the desired traces. In order to initialize the model parameters and plot the simulation traces, type *P7_2cmd* at the Matlab command prompt.

P7.3
In the previous problem, the gains of the PID position controller are set to provide a strictly aperiodical response. The model does not contain nonlinear elements and operates in linear mode. Notwithstanding linear operation and aperiodic settings, the output position overshoots the target when K_P is in the direct path. Provide the sample transfer function $W_{SS}(s)$ without conjugate complex poles or zeros, yet resulting in a step response comprising an overshoot.

P7.4
Use the previous Simulink model with the proportional gain in the direct path. Replace the position reference generator with a *repeating sequence* block and make an attempt to reduce the overshoot in the output position

by smoothing the ramp profile at its starting and ending regions. Use Matlab files *P7_4cmd.m* and *P7_4.mdl*.

P7.5

Use the previous Simulink model and replace the proportional gain into the feedback path. Introduce the incremental implementation of the proportional control action, in accordance with CH6_Fig. 1. (*Fig* 1 *given at the bottom of CH6 Exercises, see previous PDF file*) Add the feedforward compensation $y_{FB} = K_{FB}K_P(\theta^*_n - \theta^*_{n-1})$ to the summation junction providing the signal Δy_1. Run the model and investigate the impact of the feedforward gain on the tracking error. Use Matlab files *P7_5cmd.m* and *P7_5.mdl*. Hint: enter the string

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>> ff = KP/2; p7_5cmd
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in order to obtain the output position and the tracking error obtained with the feedforward gain of $K_P/2$.

P7.6

The position reference profile is given by seven time-position data pairs: $\theta(0) = 0$ rad, $\theta(0.05) = 0.002$ rad, $\theta(0.1) = 0.006$ rad, $\theta(0.15) = 0.011$ rad, $\theta(0.2) = 0.014$ rad, $\theta(0.25) = 0.016$ rad, and $\theta(0.3) = 0.017$ rad. By using linear interpolation, calculate the reference trajectory with the time resolution of 1 ms. Use the Matlab command *interp1*(). Analyze the first and second derivative of the generated trajectory.

P7.7

Consider the position reference profile given in the previous problem. Use the Simulink model of the PID position controller, contained within the Matlab file *P7_7.mdl*, to obtain the output position and torque responses. Compare the traces obtained with the profile defined in seven points (*stim*, *sdat*) and the traces obtained with linear interpolation (*stim1*, *sdat1*). Hint: use the Matlab command file *P7_7cmd.m*.

P7.8

Repeat the simulation described in the previous problem with the reference profile obtained with cubic spline interpolation. Compare the output position and torque waveforms obtained with linear and spline interpolation. Hint: use the Matlab command file *P7_8cmd.mdl*.

P7.9

Consider the reference profile *sdat1* obtained by cubic spline interpolation in P7.8. Use the Matlab *diff*() command to probe the first, second, and third derivatives.

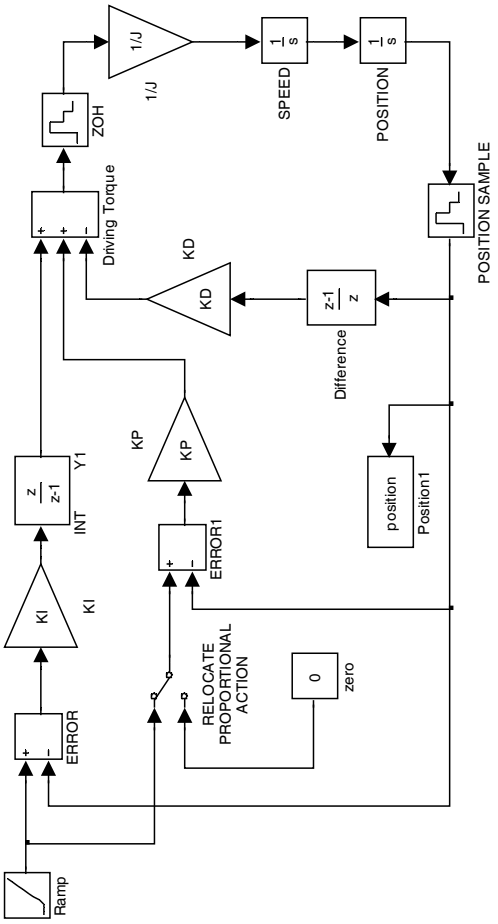

Fig. 7.2. Simplified Simulink model of linear discrete-time PID position controller. Parameters K_{FB} and K_M are assumed to be equal to one. The switch entitled *Relocate proportional action* can be operated to replace the proportional control action from the direct path into the feedback path.