Exercise #8

Anti resonant filters

P8.1
A mechanical resonator consists of two rigid bodies, each one having inertia J = 0.001 kgm^2. They are coupled by means of a flexible shaft, with stiffness K_K = 500 Nm/rad. The internal (viscous) friction of the shaft is K_V = 0.01 Nm/(rad/s). Calculate the resonant and antiresonant natural frequency and relevant damping factors. (Hint: use Eq. 8.5, see bottom of this file.)

P8.2
The mechanical subsystem in the previous problem is used as the control object of a speed-controlled system. Derive the transfer function $W_P(s)$ = $\omega(s)/T_{em}$(s) and use Matlab in order to obtain the transient response of the output speed to the torque impulse. Compare the responses obtained with K_V = 0.01 Nm/(rad/s) and K_V = 0.1 Nm/(rad/s).

P8.3
Consider a speed-controlled system with a PI controller, where the feedback gains K_P and K_I are set to their optimized values, according to the design rule derived in (Exercises, CH4). The sampling time of the system is T = 300 µs, the motor and load inertia J_M = J_L = 0.0007 kgm^2, the shaft parameters K_K = 5000 Nm/rad and K_V = 0.15 Nm/(rad/s), and the system parameters K_{FB} and K_M are equal to one. What is the resonant frequency of the torsional resonance modes? Use Simulink to obtain the step response of the output speed. Reduce the closed loop gain in order to obtain a stable, acceptable response of the output speed. Hint: use the model file *P8_3.mdl*, the command file *P8_3cmd.m*, and perform the gain reduction in steps such as K_{PNEW} = K_P/2 and K_{INEW} = K_I/4.

P8.4
Consider the step response of the output speed obtained in the previous problem with K_P = K_{POPT}/4 and K_I = K_{IOPT}/16. Estimate the rise time and the closed-loop bandwidth. Calculate the ratio f_{TR}/f_{BW} and verify the *rule of thumb* devised by

A rule of thumb>> the elasticity in mechanical coupling and the associated torsional resonance phenomena can be neglected in cases where the resonant frequency exceeds the target bandwidth by a factor of $f_{TR}/f_{BW} > 7$.

P8.5

In problem P8.3, where the range of applicable gains is sought by reducing K_P and K_I according to the formula $K_{PNEW} = K_P/2$ and $K_{INEW} = K_I/4$, the ratio K_P/K_I is not preserved. Find the explanation for this decision. (Hint: consider the impact of the gains on the natural frequency and damping of the closed loop poles. Refer to the s-domain simplified representation of the speed-controlled system, given in *Exercises_CH2_Figure….*).

P8.6

The transfer function $W_{RR2}(s)$ is obtained in Eq. 8.10, comprising a pair of weakly-damped conjugate complex zeros and one real pole. Confirm the assumption that zeros in Eq. 8.10 do not contribute to oscillations in the driving torque and rotor speed. Suggestion: in order to resolve the problem of the transfer function $W_{RR2}(s)$ being improper, extend the transfer function with the inertial load $1/Js$. Consider the values of $\omega_z = 1$, $2\xi/\omega_z = 0.01$, and $J=1$.

P8.7

Design a continuous-time notch filter with two conjugate complex poles and two conjugate complex zeros, with a notch frequency of $\omega_{NF} = 1$ rad/s, with the damping of zeros $\xi_z = 0.001$ and with the damping of poles $\xi_p = 0.5$. Calculate the discrete-time equivalent of the filter, assuming that $T = 1$. Verify the amplitude characteristic of the filter by feeding the noise signal to the input and observing the spectrum of the filter output. In order to use the prescribed approach and obtain an estimate of the amplitude characteristics, the spectral energy of the noise must have a uniform distribution.

P8.8

Design the discrete-time notch filter applied as a series antiresonant compensator in a speed-controlled system. The system parameters are $J_M = J_L = 0.001$ kgm^2, $K_K = 500$ Nm/rad, $K_V = 0.01$ Nm/(rad/s), and $T = 100$ μs. The damping of the notch filter poles is $\xi_P = 0.2$. The notch filter implementation details are given by:

$$W_{NF}(z) = \frac{K_3 z^2 - K_4 z + K_5}{z^2 - K_1 z + K_2}.$$

The coefficients K_1, K_2, K_3, K_4, and K_5 of the pulse transfer function can be obtained from its poles $z_{p1/2}$ and zeros $z_{z1/2}$. The notch poles and zeros in the s-domain are obtained as $s_{p1/2} = -\xi_P\omega_{NF} \pm j\,(1-\xi_P^2)^{1/2}\,\omega_{NF}$ and $s_{p1/2} = -\xi_p\omega_{NF} \pm j\,(1-\xi_p^2)^{1/2}\,\omega_{NF}$. In order to cancel the resonance poles, the notch frequency ω_{NF} and damping of zeros must be set to $\xi_Z = \xi_p$ and $\omega_{NF} = \omega_p$ (Eq. 8.5). The damping coefficient ξ_P of the notch poles determines the width of the frequency band involved. If we convert $W_{NF}(z)$ polynomials in terms of $(z- z_{z1})(z- z_{z2})$ and $(z- z_{p1})(z- z_{p2})$ and calculate $z_{p1/2} = \exp(s_{p1/2}\,T_{SNF})$ and $z_{z1/2} = \exp(z_{z1/2}\,T_{SNF})$, the pulse transfer function coefficients are found as

$$
\begin{aligned}
K_1 &= 2\exp\left(-\zeta_P\omega_{NF}T_{SNF}\right)\cos\left(\omega_{NF}T_{SNF}\sqrt{1-\zeta_P^2}\right)\\
K_2 &= \exp\left(-2\zeta_P\omega_{NF}T_{SNF}\right)\\
K_3 &= \exp\left[-\left(\zeta_P-\zeta_Z\right)\omega_{NF}T_{SNF}\right]\\
K_4 &= 2\exp\left(-\zeta_P\omega_{NF}T_{SNF}\right)\cos\left(\omega_{NF}T_{SNF}\sqrt{1-\zeta_Z^2}\right)\\
K_5 &= \exp\left[-\left(\zeta_P+\zeta_Z\right)\omega_{NF}T_{SNF}\right]\ .
\end{aligned}
$$

In addition, the discrete-time notch filter can be designed by using Matlab.

The Matlab command sequence used to obtain the pulse transfer function of the discrete-time recursive notch filter.

```
>> wnf = 1000;                        % Notch frequency set to 1000 rad/s
>> xp  = 0.5;                         % Notch pole damping set to 0.5
>> xz  = 0.01;                        % Notch zero damping set to 0.01
>> Ts  = 0.0001;                      % Sampling rate set to 10 kHz
>> num = [ 1 2*wnf*xz wnf*wnf];       % Numerator of the s-domain transfer function
>> den = [ 1 2*wnf*xp wnf*wnf];       % Denominator, s-domain
>> sysd = c2d(tf(num,den),Ts)         % Conversion into z-domain and printing WNF
>> [numd,dend] = tfdata(sysd,'v')     % Obtaining the WNF(z) polynomials
```

P8.9

Consider the *FIR* filter with pulse transfer function $W(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$, supplied with an impulse at $t = 5T$. Verify using Matlab that the impact of the initial impulse vanishes in n sampling periods, where n is the order of the *FIR* filter.

$$\omega_p = \sqrt{\frac{K_K\left(J_M + J_L\right)}{J_M J_L}},\ \omega_z = \sqrt{\frac{K_K}{J_L}},$$

$$\zeta_p = \sqrt{\frac{K_V^2\left(J_M + J_L\right)}{4K_K J_M J_L}},\ \zeta_z = \sqrt{\frac{K_V^2}{4K_K J_L}}\ . \tag{8.5}$$

$$\boxed{W_{RR2}(s) = \frac{1+\frac{K_V}{K_K}s+\frac{J_L}{K_K}s^2}{1+\frac{K_V}{K_K}s} = \frac{1+\frac{2\zeta_z}{\omega_z}s+\frac{1}{\omega_z^2}s^2}{1+\frac{2\zeta_z}{\omega_z}s}.} \tag{8.10}$$