Exercise #4

Discrete time PI speed controller

P4.1
Use Matlab to obtain the step response from the transfer function $W(s) = 1/(1+s\tau)$ with $\tau = 1$s. Calculate the z-domain equivalent $W(z)$ by using the function *c2d* and assuming that the sampling time is $T = 1$s. Plot the step response obtained with the pulse transfer function $W(z)$ by means of the function *dstep*, and compare this plot to the previous one.
P4.2.
Repeat the procedure in P4.1/S4.1 with the transfer function $W(s) = (s+1)/(1+s+s^2)$.

P4.3
Consider the transfer function $W(s) = 1/(1+0.5s+s^2)$. Investigate the impact of the sampling time on the step response obtained from the discrete-time system. Use the sampling time T ranging from 0.1 s to 10 s. With $T > T_{MAX}$, the step response of the discrete-time system does not correspond to the original. Discuss the value of T_{MAX}.

P4.4
Consider the closed-loop system pulse transfer function of the discrete-time speed-controlled system with proportional gain replaced into the feedback path

Eq.

$$W_{SS}(z) = \frac{2iz^2}{z^3 - (2 - p - i)z^2 + (1 + i)z - p}$$

Use Matlab to obtain the step response with $p = 0.2027$ and $i = 0.03512$.

Use sequence of Matlab commands:

```
>> p = 0.15, i = 0.01                % Setting the p and i gains
>> den = [1   -(2-p-i)  (1+i)   -p]  % Polynomial f(z) defined as den (Eq. 4.35)
>> num =  [2*(p+i)    -2*p    0 ]    % Numerator num(z) defined as num
>> roots(den)                        % Calculates zeros of f(z) (closed-loop poles)
>> roots(num)                        % Will calculate zeros of num(z) (c.l. zeros)
>> response = dstep(num,den)         % The step response samples will be
>>                                   % obtained from the Matlab function dstep
>>                                   % and stored in the array response
>> stairs(response)                  % Plotting the step response
```

Obtain the step responses with reduced proportional gain (p = 0.1, i = 0.03512), and with reduced integral gain (p = 0.2027, i = 0.01512). Observe the response characters and overshoot. The s-domain equivalence of the PI speed controller is given in Fig. 2.2.

In light of Eq.

$$d(s) = s^2 + \frac{K_P + B}{J} s + \frac{K_I}{J} = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\Rightarrow \ \omega_n = \sqrt{\frac{K_I}{J}}, \quad \xi = \frac{K_P + B}{2J\omega_n}$$

relating the feedback gains to the natural frequency ω_n and damping ξ, discuss the results obtained from the discrete-time system.

P4.5

For the discrete-time system analyzed in P4.4, and for the parameter setting of p = 0.1, i = 0.03, and T_S = 0.001, find the equivalent closed-loop transfer function in the s-domain and obtain the step response. For the conversion from discrete time to continuous time, use the Matlab command *d2c*.

P4.6

Consider the pulse transfer function numerator *numd* and denominator *dend*, obtained in P4.5; determine the closed-loop poles and zeros in the z-domain. From the s-domain transfer function numerator *numc* and denominator *denc*, obtained in P4.5, determine the closed-loop poles and zeros in the s-domain. How are the z-domain poles and zeros related to their s-domain counterparts? Why are the closed-loop zeros different?

P4.7

Consider the pulse transfer function with numerator $numd(z) = z - 0.001$, and with denominator $dend(z) = z^2 + 0.5z + 0.8$. Assume that the sampling time is $T = 1$s. Using the Matlab command *d2c*, obtain the *s*-domain equivalent $W(s) = numc(s)/denc(s)$ by using *ZOH* and *matched* options. In both cases, verify the correspondence of the poles, zeros, and step responses.