Exercise #6

Digital PID position control

P6.1
Consider the position-controlled system in Fig. 1, comprising the PID position controller and its simplified representation, as given in Fig. 2. Assuming that the normalized feedback gains are set to $d = 0.2$, $p = 0.05$, and $i = 0.005$, and that the system parameters are $J = 0.01$ kgm^2 and $T = 1$ ms, calculate the pulse transfer function $W_{LS}(z) = \theta(z)/T^L(z)$. Use the Matlab *dstep* command to obtain the load step response. Compare the result to the traces obtained in solution S5.4 of the problem P5.4 in Chapter 5.

P6.2
For the system described in the previous problem, derive the closed-loop transfer function $W_{SS}(z) = \theta(z)/\theta^*(z)$, and obtain the output position response to the step change of the reference. Estimate the speed and the driving torque traces by calculating the derivatives of the output position. Compare the rise time to the result obtained in P5.6/S5.6.

P6.3
For the system described in the previous problem, calculate the closed-loop zeros and poles in the z-domain, and find their equivalents in the s-domain.

P6.4
Consider the system described in P6.1. Assume that the normalized feedback gains are set according to Eq. 19, $d = 0.21609$, $p = 0.0516627$, $i = 0.0052195$. Obtain the output position response to the step change of the reference. Calculate the closed-loop poles.

P6.5
The characteristic polynomial $f_{PID}(z)$ of the system given in Fig. 2 is obtained in Eq. 11. Explain why the four closed-loop poles cannot be placed in an arbitrary way.

P6.6
Consider the closed loop system transfer function $W_{SS}(s)$:

$$W_{SS}(s) = \frac{1}{\left(1+\frac{s}{\omega_p}\right)^n} .$$

Calculate the bandwidth frequency f_{BW} from the condition $|W_{SS}(j2\pi f_{BW})|$ = 1/sqrt(2). Given the value of ω_p, calculate the ratio between the bandwidth frequencies obtained with n=3 and n=4. Compare this ratio to f_{BW}^{PD}/f_{BW}^{PID}, obtained in Fig. 4.

P6.7

Consider the Simulink model of the linear discrete-time PID position controller, given in Fig. 5. The model takes into account a limited peak torque capability of the torque actuator. Assume that the load torque is equal to zero and that the position reference changes from 0 rad to 0.02 rad with an adjustable slope. The model is comprised within the file P6_7.mdl. Open the file by typing its name at the Matlab command prompt. Locate the block *repeating sequence* providing the position reference and note the parameter tx used to adjust the slope $\mathrm{d}\theta^*/\mathrm{d}t = 0.02/tx$. Run the model and observe the output position and the driving torque. Start with tx = 0.05 and decrease tx in small steps towards 0.01. Identify the maximum slope of the reference profile that provides the output position without an overshoot. Notice that the Matlab *m*-file *P6_7cmd.m* can be used to initialize the model parameters and plot the simulation traces. The search can be performed by the command below.

```
>> tx = 0.04;          % setting the desired value of tx
>> P6_7cmd             % invoking the command file that plots the sim. traces
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P6.8

The system with the linear discrete-time PID position controller operates in linear mode, provided that the driving torque and speed do not reach the system limits T_{MAX} and ω_{MAX}. With larger input and load disturbances, the system limits are reached and the system enters nonlinear operating mode. Given the system parameters K_P, K_I, T, J, and T_{MAX}, determine the largest input step $\Delta\theta^*$ that does not drive the system into nonlinear mode.

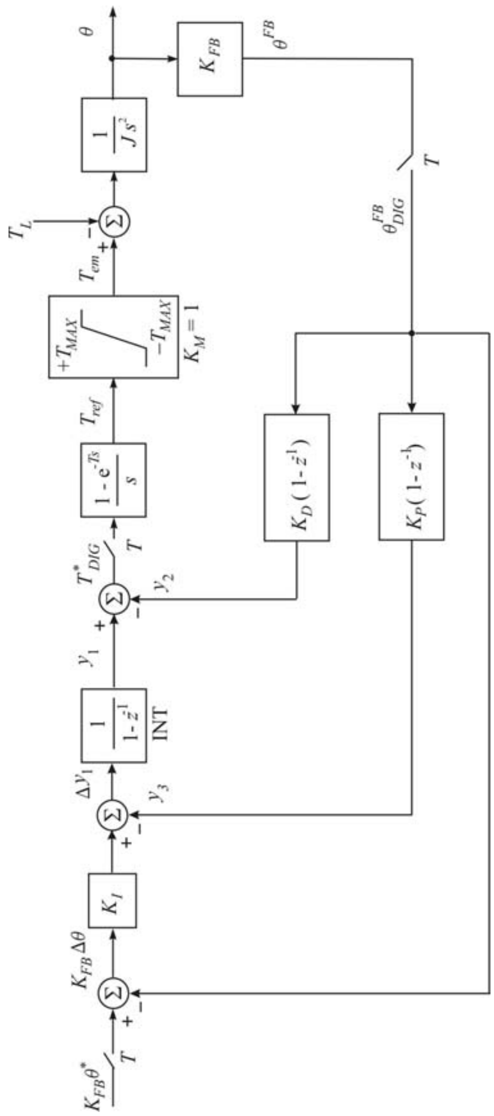

Fig. 1. Linear, discrete-time position controller with integral, proportional and derivative actions. The integral action is in the direct path and processes the tracking error $\Delta\theta$. The proportional and derivative actions are placed in the feedback path.

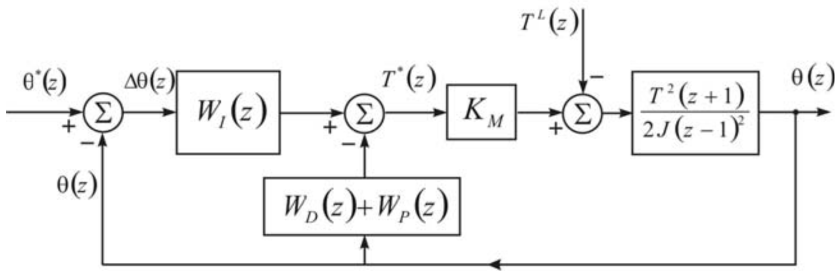

Fig. 2. Simplified block diagram of the position controller operating in linear mode. The pulse transfer functions $W_I(z)$, $W_P(z)$, and $W_D(z)$ represent the integral, proportional, and derivative actions, respectively.

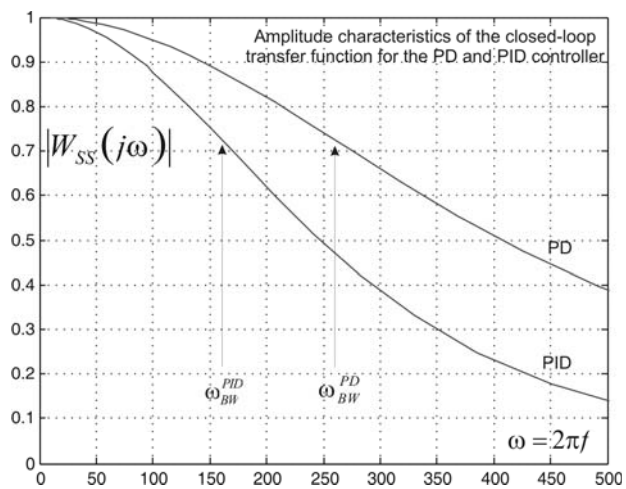

Fig. 4. The amplitude characteristics $|W_{SS}(j\omega)|$ of the PD and the PID position controller. It is assumed that the sampling time is T= 1 ms. The level of 0.707 (i.e. -3 dB) is reached for f_{BW}^{PID}=25.5 Hz and f_{BW}^{PD}=41.4 Hz.

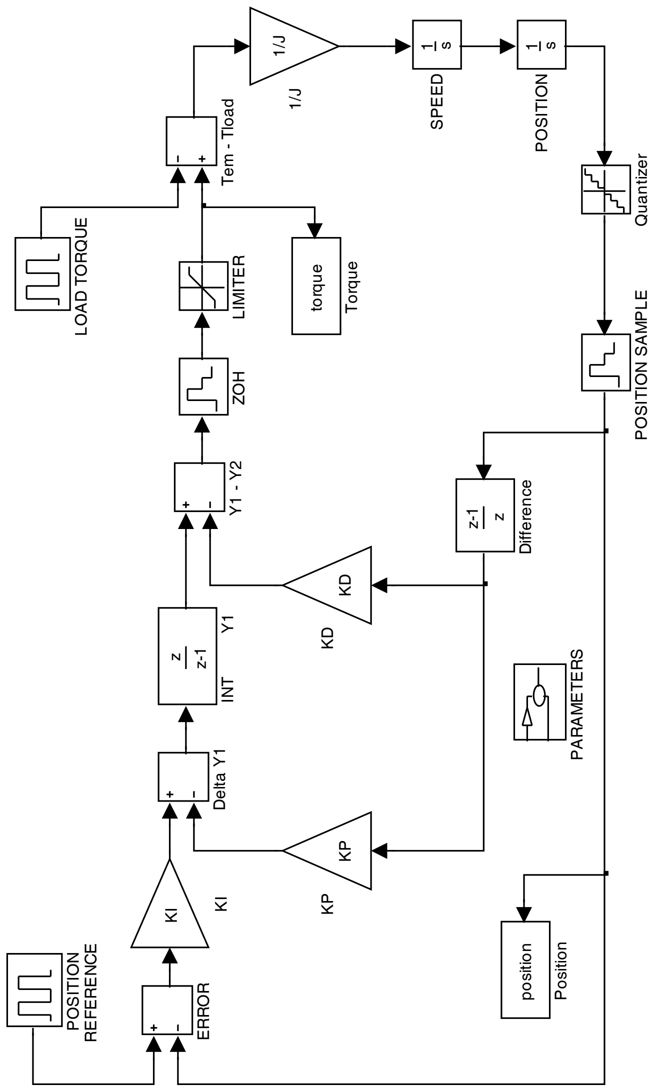

Fig. 5. Simulink model of linear discrete-time PID position controller.

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_3 = \sigma = \frac{1}{1.4667} = 0.6818 \tag{6.19}$$

$$d_{OPT} = \sigma^4 = 0.216$$

$$p_{OPT} = 4\sigma^3 - \sigma^4 - 1 = 0.0516$$

$$i_{OPT} = 6\sigma^2 + \sigma^4 - 3 = 0.0052195$$

$f_{PID}(z) = z^4 - (3 - p - i - d)z^3 + (3 - d + i)z^2 - (1 + p + d)z + d$ $= (z - \sigma_1)(z - \sigma_2)(z - \sigma_3)(z - \sigma_4)$	(6.11)