Exercise #5
Digital PD position control

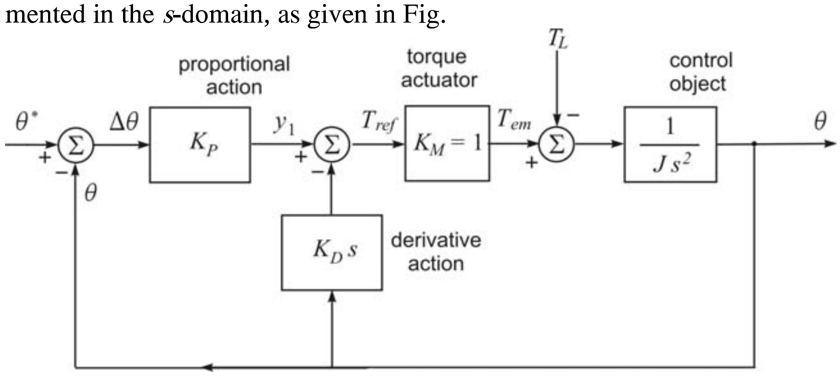
P5.1

The mechanical subsystem of a position-controlled system is described by J = 0.01 kgm^2 and B = 0.01 Nm/(rad/s). The torque actuator gain is K_M = 1. Assuming the sampling time of T = 10 ms, obtain the pulse transfer function $W_P(z)$ of the control object. Note: An appropriate sequence of Matlab commands is included:

```
>> num = [1]                      % Defines the numerator of 1/Js^2
>> den = [1  0  0]                % Denominator is 1 s^2  +  0 s  +  0
>> sysc = tf(num,den)             % Creates continuous-domain system
>> sysd = c2d(sysc,1,'zoh')       % Conversion into discrete time, T=1, ZOH;
>>                                % sysd is the discrete-time equivalent
>> tf(sysd)                       % Deriving the pulse transfer function
>> Transfer function:             %
>>     0.5 z + 0.5                %
>>  ----------------              % Matlab replies with the pulse
>>   z^2 - 2 z + 1                % transfer function  WP(z)
```

P5.2

Consider the position-controlled system with the PD controller implemented in the s-domain, as given in Fig.



Proportional-derivative position controller with the derivative action relocated into the feedback path.

The torque actuator gain is K_M = 1, the friction B has a negligible value, and the derivative gain is relocated into the feedback path. Given J = 1 kgm^2, K_P = 1 Nm/rad, and K_D = 2 Nm/(rad/s), calculate the s-domain transfer function $W_{SS}(s)$ of the closed-loop system (hint: Eq. $W_{SS}(s) = \dfrac{\theta(s)}{\theta^*(s)} = \dfrac{K_P}{K_P + K_D s + Js^2} = \dfrac{1}{1 + s\dfrac{K_D}{K_P} + 1 + s^2\dfrac{J}{K_P}}$)

Obtain the output position transient response to the reference step by using Matlab. Determine the rise time and estimate the closed-loop bandwidth.

P5.3

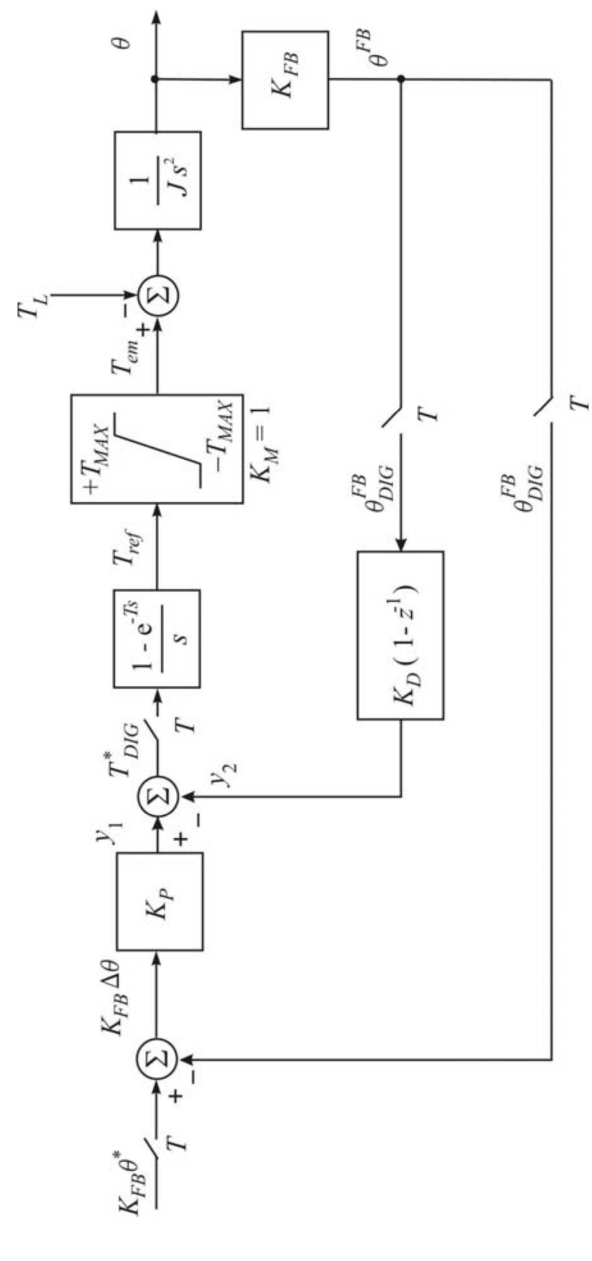
Use the previous example to investigate the impact of the parameter K_D on the closed-loop step response. What is the value of K_D that results in an overshoot of 50%? What happens when the derivative gain is completely removed?

P5.4

Consider the system described in P5.2. Calculate the transfer function $W_{LS}(s) = \theta(s)/T_L(s)$. Assuming that the load torque exhibits a step change $T_L(t) = T_{LOAD}\, h(t) = 5\, h(t)$ Nm, obtain the Laplace transform of the output position $\theta(s)$ and derive the corresponding $\theta(t)$ by using Matlab. What is the steady-state position error? Compare this result to Eq.

$$\theta(\infty) = \lim_{s \to 0}\left(s\,\theta(s)\right) = \lim_{s \to 0}\left(s\frac{T_{LOAD}}{s} W_{LS}(s)\right) = -\frac{T_{LOAD}}{K_P}$$

P5.5
Consider the position controlled system in Fig. below:



The discrete-time position controller has the proportional gain in the direct path and the derivative gain in the feedback path. The signals internal to the controller, such as $\Delta\theta$, y_1, T_{ref}, and θ^{FB}, are digital words residing within the DSP controller RAM memory. These signal have no units. What are the units of K_{FB}, K_M, K_P, and K_D? Consider the case where J = 0.01 kgm^2, T = 1ms, with K_{FB} = 1 [] and K_M = 1 []. Determine the optimized gains resulting in the fastest strictly aperiodic response.

P5.6

Use Matlab and the closed-loop transfer function of the position controlled system with the PD controller given in Eq.

$$W_{SS}(z) = \frac{z(z+1)\,p}{z(z-1)^2 + [d\,(z-1) + pz](z+1)}$$

$$= \frac{pz^2 + pz}{z^3 - (2-p-d)\,z^2 + (1+p)\,z - d}$$

to obtain the response of the output position to the step change in the reference position. Use the *dstep* command to obtain the output position and the *diff* command to obtain the torque waveform. Assume that normalized gains are set to p = 0.03512 and d = 0.2027. What is the rise time expressed in terms of the sampling period?

P5.7

The position-controlled system in Fig. (see P5.5) has a torque actuator with a peak torque of T_{MAX} = 10 Nm and an inertia of J = 0.01 kgm^2. The system approaches the target position at a speed of ω_1. At a given instant, the remaining path $\Delta\theta$ is equal to 100 rad. Considering the fact that the braking torque is limited, what is the maximum value of ω_1? How is it related to $\Delta\theta$?