# ALGORITHM FOR SUPRESSING THE MECHANICAL RESONANCE IN HIGH PERFORMANCE SERVO DRIVE

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**Abstract -** Mechanical resonance in modern servo drive systems is the primary obstacle in extending the servo loop bandwidth. To enable the extension of the range of stable gains in the presence of flexible coupling, a simple and straightforward modification of the speed loop controller is proposed in order to augment the performance of present speed controllers, and to make them accordant with mechanical structure suffering from the compliance problem. The paper comprises the analytical considerations, straightforward design guidelines, and results of experimental verification obtained by an experimental setup.

## 1. INTRODUCTION

Modern machining centers require electrical servo actuators with very high precision and the speed of response. The bandwidth of speed control loop required by NC machine tools, punching, water- and laser-cutting machines exceeds 100-200 Hz. Elastic couplings and joints within the system plant are major impediments to the performance enhancement, since high loop gains often destabilize torsional resonance modes associated with the transmission flexibility.

Servo motors in a typical industrial environment are linked to their end effectuators by transmission mechanisms having a finite stiffness. The elastically coupled two-mass motor/load system introduces finite zeros and the pair of conjugate complex poles in the transfer function of the system plant and thus brings up the problem of mechanical resonance. The problem is more emphasized in servo systems with position feedback sensors attached to the load side, e.g., to the end effectuator. Then the closed loop will encompass unmodelled torsional resonance modes. The same resonance phenomenon appaers in servo systems with a feedback sensor attached to the motor shaft. In such a case, the range of stable loop gains is severely limited along with the overall drive performance. Moreover, the load impact or the reference step input may provoke weakly damped oscillations of the link.

Often ignored in designing conventional servo systems, the resonance modes might overlap with the bandwidth of speed control loop and cause sustained oscillations of the machine links and parts. This behavior is often perceived as an audible 'humming' of the system, caused by (100 - 300) Hz oscillations of the system parts and masses. The noise is accompanied by excessive torque/force oscillations that might damage joints and parts. Besides the metal fatigue, parasitic oscillations leave their signature on the work piece, in the case of machine tool, wood-working, laser- and water-cutting machines.

High servo loop gains are required to decrease the error in tracking of fast and steep trajectories. Nevertheless, high control loop gains cannot be practiced due to mechanical resonance modes frequently encountered as unmodelled dynamics within the servo system plant. This paper provides means of improving robustness of existing servo loop controllers with respect to torsional resonance, enabling significant increase of stable control loop gains.

In the field of industrial drives, the torsional resonance problems were encountered first in rolling mill applications [1], [2], [3]. Long shafts and large inertia constitute a weakly damped mechanical resonator exhibiting a relatively low (10 - 20) Hz resonance frequency. Vibrations caused by the load impact and the step input endanger the integrity of mechanical structure and deteriorate the product quality. Most drive systems possess single feedback device and oscillatory mechanical subsystem with states that are generally unmeasurable. For that reason, the suppression of mechanical vibrations is a control problem that attracted the attention of many researchers over the past decade. Solutions being proposed may be divided into three groups: (i) Control strategies based on the direct measurement of motor- and load-side variables [5] (ii) Strategies involving only one feedback device attached to the motor and the observer that estimates remaining states [2],[3],[4],[6]. (iii) Vibration suppression strategies based upon the notch filtering and phase-lead compensation applied in conventional speed control structures [4], [7].

The noise contaminating detected speed and position signals hasn't the properties the Gaussian noise; it rather contains both components related to the PWM frequency of the drive amplifier and variable frequency components related to the motor speed, number of motor and resolver poles, and resolver excitation frequency. Thus, state variables of a mechanical subsystem can be estimated from detected signals with a limited bandwidth and relatively large transport lags that are inappropriate for suppressing unwanted mechanical oscillations above 100 Hz. *The desired speed-loop bandwidth in modern machining centers approach the frequency of torsional resonance, and coincide at the same time with most disturbing statistical and deterministic noises.* An estimation of the torsional torque under aforementioned conditions becomes difficult, not promising exact and sufficiently fast estimates.

We propose a simple software implemented filtering structure, which extends the range of applicable gains that significantly enhance servo performances. The proposed structure is intended for high performance servo drives with the speed loop bandwidth above 100 Hz range, where the esimation of unmeasured states is not feasible. The antiresonant feature of the structure is not based on the exact cancellation of resonance poles and is, therefore, robust against parameter fluctuations and easy to use.

2.SERVO SYSTEM WITH ELASTIC DRIVE TRAIN

Two-mass motor/load system with flexible coupling is shown in Fig. 1. The electromagnetic torque $M_{em}$ is obtained at the output of speed controller. The motor inertia $J_m$ and load inertia $J_l$ are coupled by the transmission system (in majority of cases, shaft, toothed belt, or gear transmission) having a finite stiffness coefficient $K_s$. A flexible coupling doubles the number of state variables within the mechanical subsystem of the drive. Generally, the speed $\omega_m$ and position $\theta_m$ of the motor shaft differ from the respective variables $\omega_l$ and $\theta_l$, on the load side. The two-mass $J_m$ - $K_s$ - $J_l$ system is damped mainly by the viscous friction $K_v\Delta\omega$. The friction coefficient $K_v$ generally assumes very low values, giving rise to weakly damped mechanical oscillations. The torsional torque $M_o$ equals the load torque $M_l$ only in the steady-state. During transients, speeds of motor and load differ, and torsional torque $M_o$ is given by (1).

$$M_o = K_s \Delta\theta + K_v \Delta\omega \,. \qquad (1)$$

$$W_m(s) = \frac{1}{(J_m + J_l)s} \; \frac{1 + \frac{2\zeta_z}{\omega_z} s + \frac{1}{\omega_z^2} s^2}{1 + \frac{2\zeta_p}{\omega_p} s + \frac{1}{\omega_p^2} s^2} \,. \qquad (2)$$

$$W_l(s) = \frac{1}{(J_m + J_l)s} \; \frac{1 + \frac{2\zeta_z}{\omega_z} s}{1 + \frac{2\zeta_p}{\omega_p} s + \frac{1}{\omega_p^2} s^2} \,. \qquad (3)$$

Transfer function $W_m(s) = \omega_{out}(s)/M_{em}(s)$ of the mechanical subsystem differs from the simple $1/Js$. If the shaft sensor is mounted on the motor, $W_m(s)$ is defined by (2). For systems where the shaft sensor detects load variables $\omega_l$ and $\theta_l$ at the end effectuator, the mechanical subsystem of the drive has the transfer function $W_l(s)$ given by (3). In (2) and (3), undamped natural frequencies ($\omega_p$ ,$\omega_z$,) and relative damping coefficients ($\zeta_p$ ,$\zeta_z$ ) are given by (4).

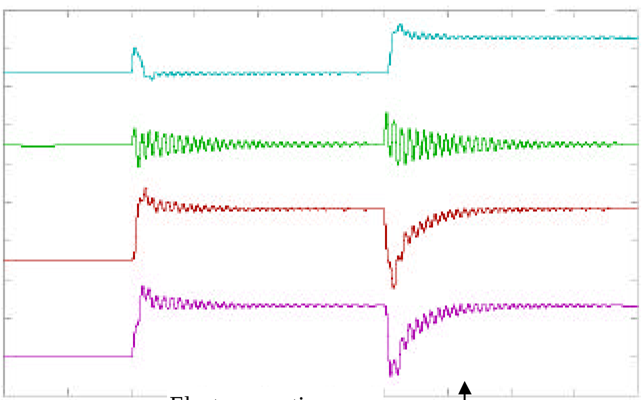
$$\omega_p = \sqrt{\frac{K_s (J_m + J_l)}{J_m J_l}}, \quad \omega_z = \sqrt{\frac{K_s}{J_l}},$$

$$\zeta_p = \sqrt{\frac{K_v^2 (J_m + J_l)}{4 K_s J_m J_l}}, \quad \zeta_z = \sqrt{\frac{K_v^2}{4 K_s J_l}}. \qquad (4)$$

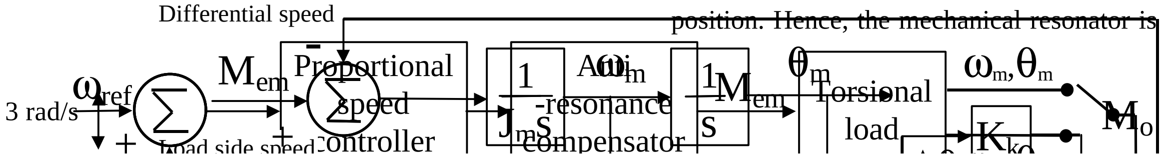
The effect of flexible coupling on the closed loop performance of the drive is strongly influenced by location of the feedback transducer. Fig. 2 shows the block diagram of a simple proportional speed controller applied to the motor with torsional load. The block diagram indicates the possibilities of closing the control loop by using either motor- or load-side feedback and application of a cascade compensator that cancels or mediates unwanted resonance modes associated with a flexible mechanical subsystem. In the later analysis, related to the structure in Fig. 2, the drive system is considered ideal except for the time lag caused by the speed calculation, control algorithm execution, and dynamics of the stator current controller.

Undamped natural frequencies $\omega_p$ and $\omega_z$ of pole- and zero-pairs in Eq. (2) are referred to as the resonance and anti-resonance frequencies [4], and their quotient is known as the resonance ratio :

$$R_r = \frac{\omega_p}{\omega_z} = \sqrt{1 + \frac{J_l}{J_m}} \,. \qquad (5)$$




Attached to the motor, the sensor provides correct motor speed signal; that is, the motor-load compliance does not deteriorate measurement of the motor speed and position. Hence, the mechanical resonator is not enclosed in



the control loop. A low value of resonance ratio reduces the influence of torsional load on the dynamics of speed control loop. With $J_m >> J_l$, oscillations of torsional torque are filtered by a large motor inertia $J_m$ and their influence on the control of the motor speed becomes smaller.

Nevertheless, the case $J_m >> J_l$ with resonance ratio $R_r$ close to 1 is most critical as far as the performance in control of load side variables is of concerned. Precisely, damped control of $\theta_m$ and $\omega_m$ is favorable, but most applications require fast and precise control of the end effectuator and the load variables $\theta_l$ and $\omega_l$. Since a large motor inertia ($J_m >> J_l$) impedes the penetration of torsional oscillations from the load to the motor, the estimation of resonance modes from detected signals ($\theta_m$ and $\omega_m$) is not feasible. Actually, the motor speed $\omega_m$ contains information on remote states ($\theta_l$ and $\omega_l$) of mechanical subsystem, though in an amount commensurable with the ratio $J_l / J_m$. Moreover, in the case of $J_m >> J_l$, secondary effects (the noise and sensor imperfection) exclude the possibility of applying an observer of load side states. Therefore, even with a stiff and rigid control over $\omega_m$,, the load speed and position might exhibit weakly damped oscillations that cannot be disclosed and compensated from the feedback signals.

## 3. ANTI-RESONANCE NOTCH SERIES COMPENSATOR

Mechanical resonance is encountered in all cases with low value of damping coefficiend (see equation 4). The task of increasing the relative damping coefficient of critical poles might be accomplished by the cascade compensator with the notch filter. The notch filter zeros (Eq. 5) are to cancel critical poles, while the poles of the filter become a new pair of conjugate complex poles with increased relative damping coefficient ($\zeta_p >> \zeta_z$). The notch filter, implemented as shown in Eq. 6,

$$W_{notch} = \frac{s^2 + 2\zeta_z \omega_{NF} s + \omega_{NF}^2}{s^2 + 2\zeta_p \omega_{NF} s + \omega_{NF}^2}. \tag{5}$$

$$W_{NF}(z) = \frac{K_3 z^2 - K_4 z + K_5}{z^2 - K_1 z + K_2}. \tag{6}$$

requires precise setting of five parameters K1-K5, listed in Eq. 7. When correctly tuned and inserted as the anti-resonance block in Fig. 2, the notch filter prevents the energy flow from the servo amplifier into the motor and the system mechanics at the center frequency of the band-stop zone. In such a way, the resonance modes of the mechanical subsystem will receive no excitation and responses will be smooth.

In the mechanical subsystem, the equivalent inertia changes during each operating cycle, and the friction coefficient varies with the speed and machine wear. Hence, the exact location of critical poles is unknown, and thus the cancellation is generally imprecise.

$$\begin{aligned}
K_1 &= 2\exp(-\zeta_p \omega_p T)\cos(\omega_p T\sqrt{1-\zeta_p^2}),\\
K_2 &= \exp(-2\zeta_p \omega_p T),\\
K_3 &= \exp[-(\zeta_p - \zeta_z)\omega_p T],\\
K_4 &= 2\exp(-\zeta_p \omega_p T)\cos(\omega_p T\sqrt{1-\zeta_z^2}),\\
K_5 &= \exp[-(\zeta_p + \zeta_z)\omega_p T].
\end{aligned} \tag{7}$$

Then, the resonance poles and the zeros of notch filter will not coincide and pole-zero doublets will appear in the root locus. The notch compensation, even mismatched, might significantly increase the range of stable gains. Nevertheless, the robustness of notch compensation with respect to a parameter mismatch is bounded to +/- 25% range around the nominal values: while the resonant frequency $\omega_p$ may be obtained off-line or on-line without major difficulties, the viscous friction is generally not known, and it sets a serious problem in tuning the notch compensator. Hence, a detuning of filter parameters K1-K5 (Equation 7) greatly reduces positive effects of the compensator.

The effects of an anti-resonant notch filter within speed servo loop (Fig. 2) are investigated experimentaly. Two identical motors are connected via elastic hollow shaft. The motors are independently controlled and used as the motor and the load (Figure 1). The presence of electromagnetic resolver at each end enabled the testing of the motor- and the load-side feedback modes. The motor data are $T_{nom}$ = 5.7 Nm (the rated torque), $T_{max}$ = 24 Nm, $\omega_{nom}$ = 3000 rpm, $P_{nom}$ = 1.49 kW, $J$ = 0.000620 $kgm^2$ (inertia of one motor including resolver), hollow shaft parameters: $J_{sh}$ = 0.000220 $kgm^2$ (the shaft and the coupling inertia) $K_s$ = 350 Nm/rad (stiffness), $K_v$ = 0.004 Nms/rad (viscous friction). Control algorythm is implemented on the TMS320C14 DSP within the DBM03 Vickers triple axis servo amplifier. Experimental results obtained with a de-tuned notch series compensator are given in Fig. 4.

## 4. ANTI-RESONANCE FIR SERIES COMPENSATOR

We propose hereafter the cascade anti-resonance compensator conceived to be simpler, less sensitive to parameter changes, and requiring setting of only one parameter. This compensator is used instead of the notch filter of Fig. 2. to alleviate the torsional resonance problems in a passive way. The task imposed on the proposed series anti-resonant compensator is essentially to filter the torque command signal. Ideally, all spectral components should remain unaltered except those coinciding with mechanical resonance frequencies of the drive train. Hence, passive series compensator by its nature represents a band-stop filter. Consequently, as long as the driving torque has no spectral components at the resonance frequency, no energy is injected into resonance modes. In such a way, an increase of the loop gain will not sustain the energy of the mechanical resonator, and thus higher gains will be attainable along with improvements in performances of the speed control loop.

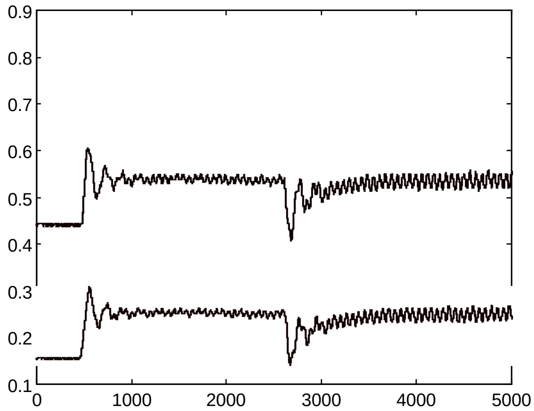



Fig. 4: Motor-side feedback with the notch series compensator. The loop gain $G$ = 100%, and 25% detuning of the notch central frequency. The experimental traces of the torque reference (upper trace 10 Nm per div.), load-side speed (middle trace 10 rad/s per div.), and the motor-side speed (lower trace).

The notch filter suppresses the resonant frequency by the ratio $\zeta_p/\zeta_z$ between the relative damping coefficients of the notch poles and zeros. Since a low damping coefficient of zeros increases greatly the sensitivity to parameter variations, the ratio $\zeta_p/\zeta_z$ is limited. Hence, the excitation of resonance modes can be only reduced, but not eliminated completely by the notch series compensator.

A simple way to fully suppress the excitation of resonance modes is to adopt the anti-resonance series compensator in the form of:

$$W_{NF}(z)=\frac{1+z^{-n}}{2};\quad n=\frac{T_{osc}}{2T_{spl}}, \tag{8}$$

where the '$n$' stands for the ratio between the resonance mode half - period and the sampling time of discrete time controller. In other words, any increment of the driving force is to be distributed into two equal steps, one of them being delayed by $nT_{spl}$. If the time delay of the second half step is set to one half period of the plant resonance frequency, subsequent oscillatory responses will have their pulsating components with opposite phase; consequently, pulsations nullify each other and the resulting response will become oscillation - free and acceptable. Experimental results obtained with proposed FIR series compensator are given in Fig. 5.

## 5. CONCLUSION

The paper proposes a simple and robust series compensator capable of extending the range of applicable gains in a modern servo drive. Basic analytical considerations are presented along with the brief overview of methods for suppressing the torsional oscillations associated with high-performance servo drives in the frequency range above 100 Hz. Primarily, the cascade notch filtering is considered as a passive way of enhancing performance of conventional speed controllers. Then, the analytical considerations and utilization of a simple FIR digital filter for the suppression of resonance oscillations are presented, along with verification by means of computer simulations. Straightforward design guidelines for tuning the proposed compensator are given. The tuning does not require plant parameters other than the half period of resonance frequency. Finally, brief explanations concerning the experimental setup are given. Measurement results are obtained on a test bed with elastically coupled 7Nm synchronous servo motor and a mechanical resonance frequency of 160 Hz. The accomplished tests demonstrate the significant improvement of servo loop bandwidth and a notable enhancement of applicable loop gains. The experimental results are in agreement with the simulation runs and fully confirm the validity and efficiency of the proposed solutions.

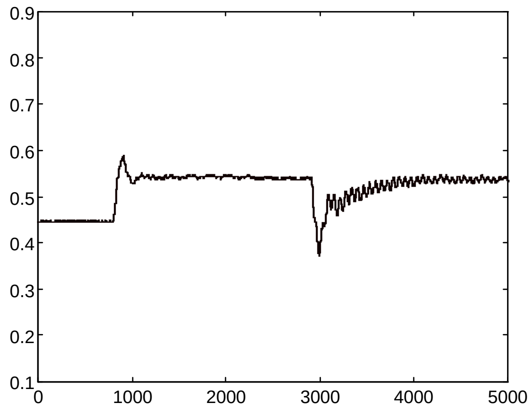



Fig. 5: Motor-side feedback with the FIR series compensator. The loop gain $G$ = 100%, and 25% detuning of compensator's central frequency. The experimental traces of the torque reference (upper trace 10 Nm per div.), load-side speed (middle trace 10 rad/s per div.), and the motor-side speed (lower trace).

## LITERATURE

[1] Ohmae, T. Matsuda, M. Kanno, K. Saito, and T. Sukegawa, "A Microprocessor- based motor speed regulator using fast-response state observer for reduction of torsional vibrations," *IEEE Trans. on Ind. Applications*, vol. IA-23, no. 5, pp. 863-871, Sep/Oct. 1987.

[2] R. Dhauadi *et al*., "Robust speed control of rolling mill drive systems using the loop transfer recovery design methodology," in *Proceedings of the IEEE IECON 1991*, pp. 555-560.

[3] R. Dhauadi *et al*., "Two degree of freedom robust speed controller for high performance rolling mill drives," in *IEEE IAS Conf. Rec*. *1992*. pp.. 400-407.

[4] K. Sugiura and Y. Hori, "Vibration suppression in 2- and 3-mass system based on the feedback of imperfect derivative of the estimated torsional torque," *IEEE Trans. Ind. Electronics*, vol. IE-43, no. 1, pp.. 56-64, Feb. 1996.

[5] J.Y.Hung, "Control of industrial robots that have transmission elasticity," *IEEE Trans. Ind. Electronics*, vol. IE-38, no. 6, pp. 421-427, Dec. 1991.

[6] J.K. Ji and S. K. Sul, "Kalman filter and LQ based speed controller for torsional vibration suppression in a 2-mass

motor drive system," *IEEE Trans. Ind. Electronics*, vol. 42, no. 6, pp.. 564-571, Dec. 1995.

[7] F. Ghorbel, J.Y. Hung, and M.W. Spong, "Adaptive control of flexible joint manipulators," *IEEE Control Systems Magazine,* vol. 9, no.7, pp. 9-13, Dec. 1989.